GAME THEORY

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Game Theory

Overview

Game theory is a branch of logic which deals with cooperation and conflict in the context of negotiations and payoffs. The theory of games can elucidate the incentive conditions required for cooperation, can aid understanding of strategic decisions of nations or actors in conflict, and can help in the development of models of bargaining and deterrence.

Key Concepts and Terms

The Prisoner's Dilemma

The "prisoner's dilemma" is the classic game in game theory literature. It centers on a game in which both actors would be better off cooperating, but both have an individual incentive to defect (not to cooperate) and as a result the likely outcome is one which is worse for both players than had they cooperated. This is illustrated in Table I below.

Repeated games

In real life, most games are repeated rather than single-shot. Repetition means each player has additional information based on past game decisions of the other player. This complicates calculation of choices and changes the equilibrium point. See Fink, Gates, and Humes, 1998: ch. 3. For instance, if the Prisoner's Dilemma is repeated a sufficient number of times, for instance, players may learn to take a strategic view and cooperate.

Strategies

A strategy is a plan of action that cannot be upset by an opponent or nature. In the prisoner's dilemma, the options are "confess" and "do not confess," and in one-round games, the strategy is the option taken. In multi-round games, strategies may be more complex. The purpose of strategies is to secure the most favorable game value in the long run. As an example, one strategy in multi-round games is *tit-for-tat*, in which the player responds to a given game move with a mirroring move.

Pure strategies

A pure strategy involves always pursuing the same strategy.

Mixed strategies

A mixed strategy involves randomly choosing among one's best strategies according to some proportions in order to maximize favorable game value when a pure strategy in repeated games would give the opponent an advantage of predictability. The part of a proportion (such as the 3 in the proportion 1:3) corresponding to a particular strategy is called the *oddment* of that strategy.

Game matrix

A game matrix is the table of all strategies of person A (as columns) versus all strategies of person B (as rows). The cell entries in a game matrix are *payoff* values, as illustrated below, with the first being the payoff to A and the second being the payoff to B. When the payoffs to strategies are quantified and placed in a matrix, the game is said to be in "strategic form."

Maximin, minimax, and saddle point

Consider the following tables:

TABLE I

Person A			
		Strategy 1	Strategy 2
Person B	Strategy 1	**-1,-1**	**-25,0**
	Strategy 2	**0,-25**	**-20,-20**

TABLE II

Person A				
		Strategy 1	Strategy 2	Strategy 3
Person B	Strategy 1	7	6	4
	Strategy 2	3	2	5

In the prisoner's dilemma, let Strategy 1 b "Not confess" and Strategy 2 be "Confess". Assume each person considers only payoffs to him/herself, not payoffs to the other player. With full information and full trust, the solution would be for each person to choose Strategy 1, causing each person to serve only one year in jail. However, using conservative self-interested reasoning, Person B will select the strategy where the least to be gained is highest, or in this example, the most to be lost is lowest. The row containing the highest minimum (the maximum row minimum, or *maximin*) for Person B in Table I is the row for Strategy 2, where the maximin is minus 20. That is, the with Strategy 2 the worst Person B will get is 20 years in jail, whereas with Strategy 1 the worst is 25 years. Using conservative reasoning, Person A will also select Strategy 2.

Zero-sum games

Table II above-right shows a different formatting for a game matrix. In this format, only the payoffs to one player are shown, by convention the second player, B. Th game is "zero sum" because gain to one player, B, is seen as loss to the other player, A. The objective is assumed to be for Player B to maximize his/her payoff and for Player B to seek to minimize the payoff to A. Here the payoffs are positive, meaning higher values are desirable. By conservative strategy, Player B will select the maximin, which is Strategy 1, where the maximum row minimum is 4. Player A in Table II above-right will seek the column containing the lowest maximum (the minimum column maximum, or minimax), which is Strategy 3, where the minimum column maximum is 5.

A zero-sum game could also be represented with dual notation as below, with payoffs adding to zero.

TABLE II

		Person A		
		Strategy 1	Strategy 2	Strategy 3
Person B	Strategy 1	-7,7	-6,6	-4,4
	Strategy 2	-3,3	-2,2	-5,5

A *non-zero-sum game* is, of course, any game in which the payoffs do not add to zero. That is, a gain for one player is not necessarily an equal loss for the other, or even a loss at all.

Constant-sum games

Constant-sum games are very similar to zero-sum games, but the payoffs add to a constant amount, not zero. This is typical when payoffs are expressed as percentages, which add to 100% in each cell. It is a type of zero-sum game since gain in percentage to one is loss in percentage to the other.

Saddle points

When the same cell is both the maximin and the minimax (not the case above), it is the *saddle point*. Any saddle point is also the *solution* to the game because it will be the payoff which results when the game is played by opponents using conservative rationality.

Dominance and reduced games

In Table II above-right, Considering only strategies 1 and 2, Person A will always prefer strategy 2 over strategy 1. If Person B chooses strategy 1, Person A will see B gain only 6 points rather than the 7 were A to instead choose strategy 1. Likewise A will see B gain only 2 rather than 3 points if A selects strategy 2. That is, for Person A, strategy 2 *dominates* strategy 1. This means column 1 can be eliminated from the game, resulting in the matrix in Table III below. After dropping a column, the matrix is called a *reduced game.*

TABLE III

Person A				
		Strategy 2	Strategy 3	B's Odds
Person B	Strategy 1	6	4	3
	Strategy 2	2	5	2
	A's Odds	1	4	

Deadlock

Deadlock occurs when both players have a dominant strategy which results in a suboptimal outcome for both. In Table IV below, let strategy 1 be "Agree" and Strategy 2 by "Disagree." Player A will always agree because by selecting Strategy 1, Player A will get a higher payoff whatever strategy Player B selects. Likewise, Player B will always agree also because Strategy 1 results in a higher payoff whatever strategy Player B selects. Barring mistakes by the other player, neither playerwill ever get the optimal payoff of 8.

TABLE IV

		Person A	
		Strategy 1	Strategy 2
Person B	Strategy 1	**5,5**	**8,2**
	Strategy 2	**2, 8**	**3,3**

Odds in games without saddle points

In the reduced game in Table III, there was no saddle point. The maximin was 4 but the minimax was 5. Person B will prefer strategy 1 because 4 is the least to be won (and possibly 6), whereas with strategy 2 the least to be won is 2 (or possibly 5). Person A will prefer strategy 3 because the most to be lost is 5, whereas under strategy 2 the most to be lost is 6. However, person A may want to play strategy 2 once in a while if Person A is consistently playing his strategy 1 because then Person B will lose only 2, Person A can't do this consistently, however, because then Person B will move to strategy 2 and Person A will lost 5. Person A wants to usually play strategy 3, but "sneak in" a strategy 2 once in a while. The proportion for the "once in a while" is determined by the odds. To compute the odds for

Table III, subtract the cells within each column or row, ignore minus signs, and place the answer in the *opposite* column. Thus, for instance, 6 -1 =4 and 4 - 5 =-1, giving 1 and 4 for A's odds, which means A should randomly use strategy 3 once for every four times strategy 3 is used.

Fair game value

The value of a game equals either person's odds played against any single strategy of the opponent. Thus, for Table III above, [(3x6)+(2x2)]/(3+2) = 4.4. A fair game is one with a value of zero.

Side payments. In some games the payoffs are loaded in favor of one player of the other. A side payment is the amount the favored player must pay the less favored player in order for the game to be fair. In the foregoing example, B should make a *side payment* of 4.4 units to A before each game if the game is to be fair.

Other types of games

Note that there are many other types of games than zero-sum games played under conservative rationality. Assumptions about rationality may be varied, for instance, and games may be *cooperative* rather than competitively zero-sum. Also, payoffs may be ordinal rather than interval and information may not be full.

Assumptions

Game theory usually assumes players respond rationally based on payoffs in the game. The most common assumption is one of *conservative rationality,* in which players choose strategies that assume maximum average gains or minimum average losses.

Most applications of game theory assume conditions of full information, a condition rarely met in the real world.

Frequently Asked Questions

Is game theory purely a branch of logic?

No, political scientists have long been interested in a behavioral, approach to game theory, testing out the implications of formal game theory using small group experiments.

What is an extensive form game?

A game in extensive form is represented as a horizontal tree diagram. The first set of branches are alternative strategies for player 1; the second set of branches from the end nodes of the first set, are the responsive strategies for player 2; the third set of branches from the end notes of the second set are the next strategies for player 1; and so on, until the final end nodes, which are the payoff amounts for each path through the tree.

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